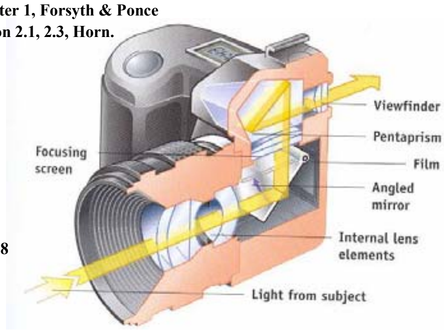
Image formation & camera basics

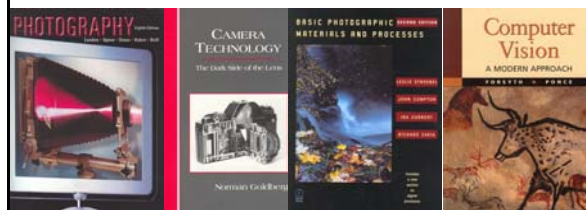
Reading: Chapter 1, Forsyth & Ponce
Optional: Section 2.1, 2.3, Horn.



February 5, 2008

Reference

- http://en.wikipedia.org/wiki/Lens_(optics)



- The slides use illustrations from these books

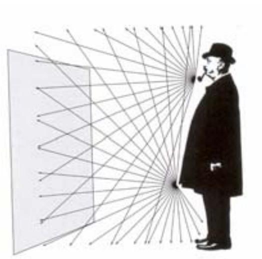
Plan

- Pinhole optics
- Lenses
- Projections
- Camera

7-year old's question

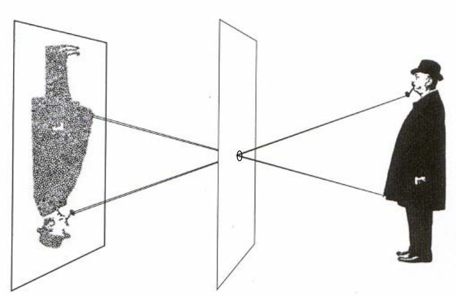
Why is there no image on a white piece of paper?

It receives light from all directions



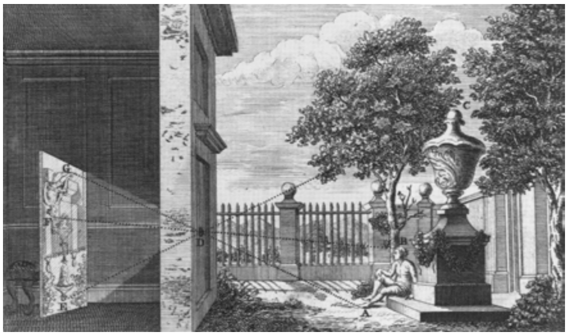
From Photography, London et al.

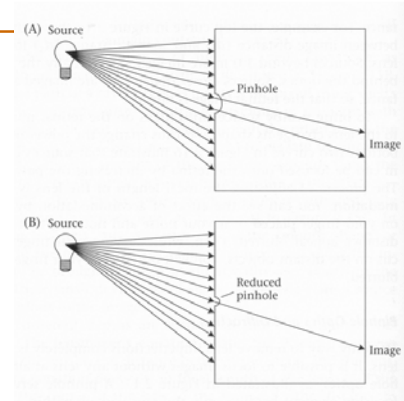
Pinhole



From Photography, London et al.

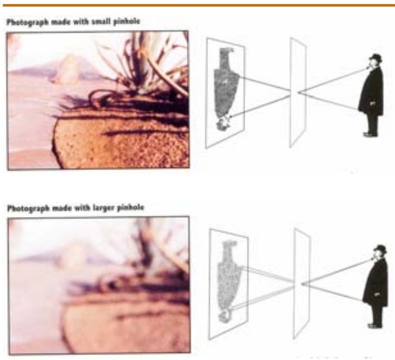
Questions?





Wandell, Foundations of Vision, Sinauer, 1995

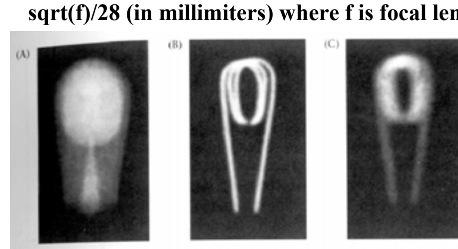
Pinhole size?



From Photography, London et al.

Diffraction limit

- Optimal size for visible light: sqrt(f)/28 (in millimiters) where f is focal length



2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

From Wandell

Problem with pinhole?

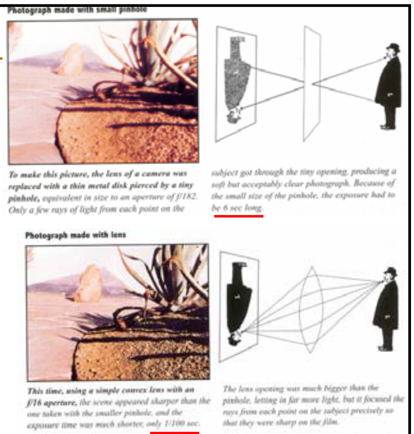
- Not enough light!
- Diffraction limits sharpness

Solution: refraction!

From Photography, London et al.

Lenses

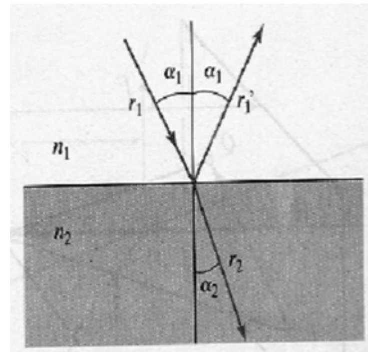
- gather more light!
- But need to be focused



From Photography, London et al.

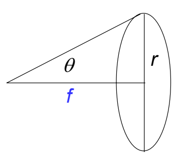
Snell's Law

$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$



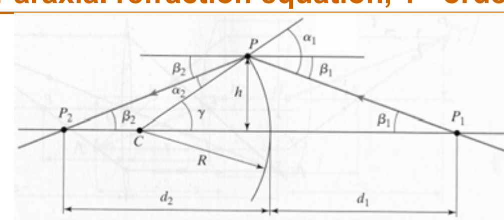
First Order Optics

$$\sin(\theta) \approx \theta$$



$$\theta = \frac{r}{f}$$

Paraxial refraction equation, 1st order



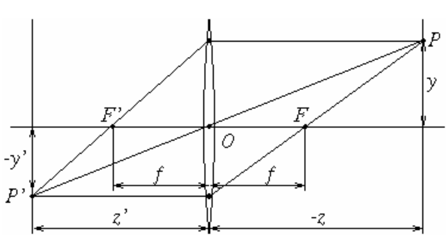
$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$

$$\alpha_1 = \gamma + \beta_1 \approx h\left(\frac{1}{R} + \frac{1}{d_1}\right)$$

$$\alpha_2 = \gamma - \beta_2 \approx h\left(\frac{1}{R} - \frac{1}{d_2}\right)$$

$$n_1\alpha_1 \approx n_2\alpha_2 \Leftrightarrow \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$$

The thin lens, first order optics

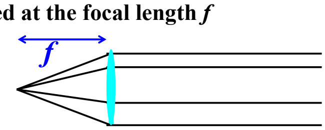


$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \qquad f = \frac{R}{2(n-1)}$$

Forsyth&Ponce

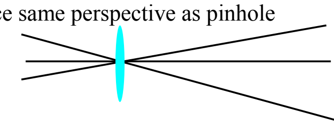
Thin lens optics

- **Simplification of geometrical optics for well-behaved lenses**
- **All parallel rays converge to one point on a plane located at the focal length *f***



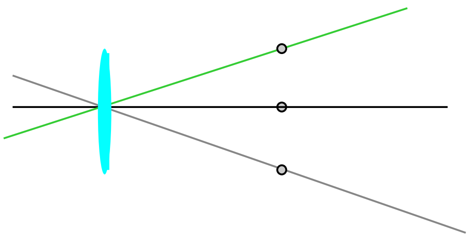
- **All rays going through the center are not deviated**

  Hence same perspective as pinhole



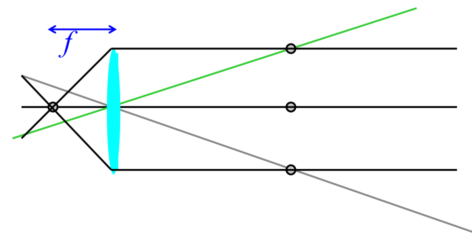
How to trace rays

- **Start by rays through the center**



How to trace rays

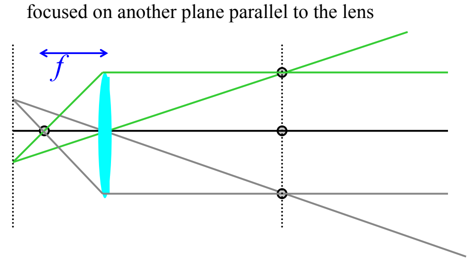
- **Start by rays through the center**
- **Choose focal length, trace parallels**



How to trace rays

- **Start by rays through the center**
- **Choose focal length, trace parallels**
- **You get the focus plane for a given scene plane**

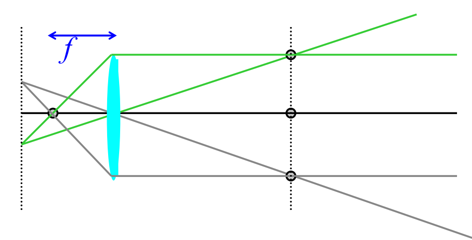
  All rays coming from points on a plane parallel to the lens are focused on another plane parallel to the lens



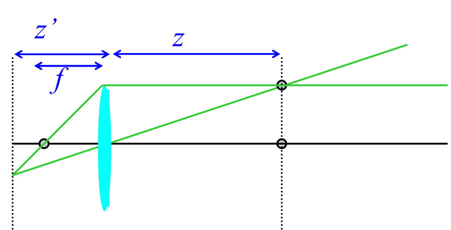
Focusing

- **To focus closer than infinity**

  Move the sensor/film ***further*** than the focal length

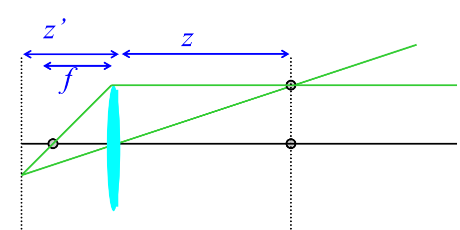


Thin lens formula



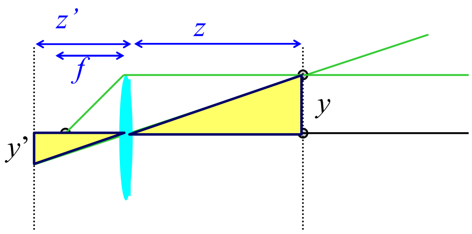
Thin lens formula

Similar triangles everywhere!



Thin lens formula

Similar triangles everywhere! $y'/y = z'/z$

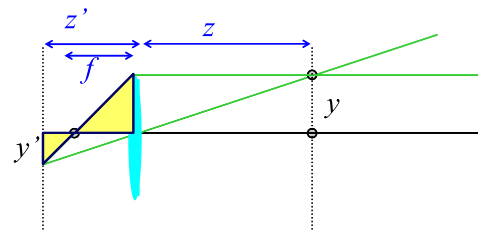


Thin lens formula

Similar triangles everywhere!

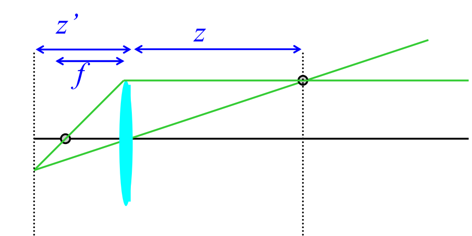
$y'/y = z'/z$

$y'/y = (z'-f)/z$



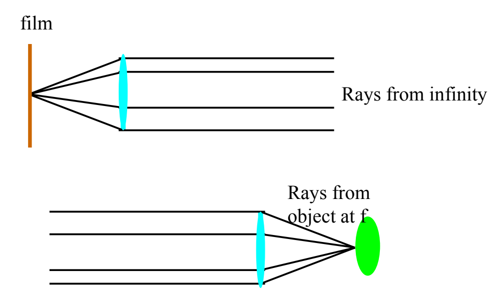
Thin lens formula

$$\frac{1}{z'} + \frac{1}{z} = \frac{1}{f}$$

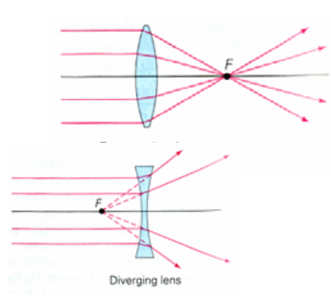


Minimum focusing distance

- **By symmetry, an object at the focal length requires the film to be at infinity.**



Convex and concave lenses



http://www.physics.uiowa.edu/~umallik/adventure/light/lenses.gif

More accurate models of real lenses

- **Finite lens thickness**
- **Higher order approximation to** $\sin(\theta)$
- **Chromatic aberration**
- **Vignetting**

Thick lens

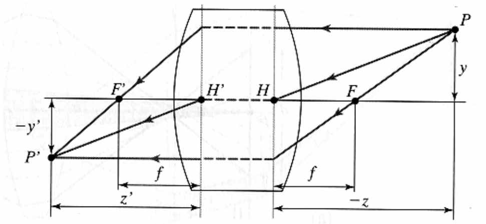
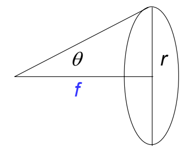


Figure 1.11 A simple thick lens with two spherical surfaces.

Third Order Optics

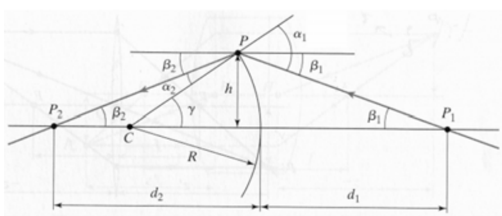
$$\sin(\theta) \approx \theta - \frac{\theta^3}{6}$$

$$r = d/2$$



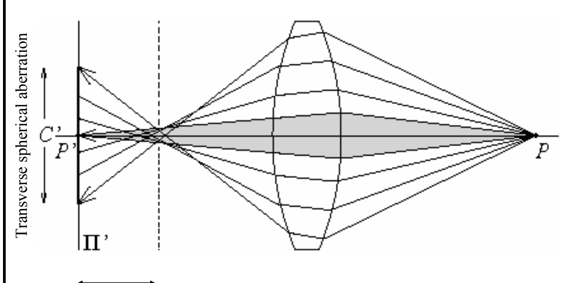
$$\theta = \frac{r}{f} - \frac{\left(\frac{r}{f}\right)^3}{6}$$

Paraxial refraction equation, 3rd order



$$\frac{n_1}{d_1}+\frac{n_2}{d_2}=\frac{n_2-n_1}{R}+h^2\left[\frac{n_1}{2d_1}\left(\frac{1}{R}+\frac{1}{d_1}\right)^2+\frac{n_2}{2d_2}\left(\frac{1}{R}-\frac{1}{d_2}\right)^2\right]$$

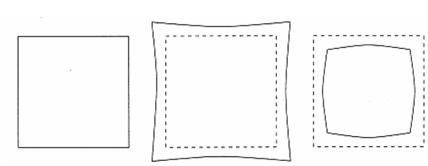
Spherical aberration (from 3rd order optics)



Longitudinal spherical aberration

Other 3rd order effects

- **Coma, astigmatism, field curvature, distortion.**

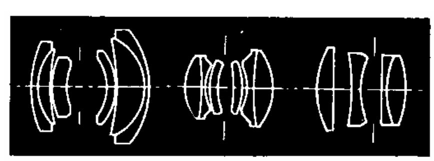


no distortion

pincushion distortion

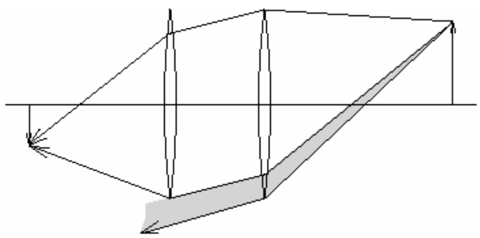
barrel distortion

Lens systems



Lens systems can be designed to correct for aberrations described by 3rd order optics

Vignetting



Other (possibly annoying) phenomena

- **Chromatic aberration**
  - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  - Machines: coat the lens
  - Humans: live with it
- **Scattering at the lens surface**
  - Some light entering the lens system is reflected off each surface it encounters (Fresnel's law gives details)
  - Machines: coat the lens, interior
  - Humans: live with it (various scattering phenomena are visible in the human eye)
- **Geometric phenomena (Barrel distortion, etc.)**

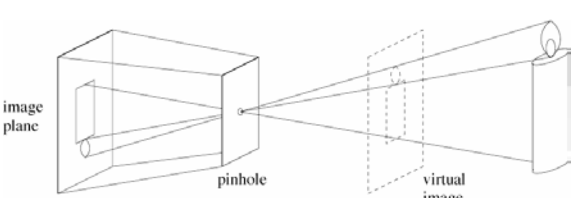
Chromatic aberration

(great for prisms, bad for lenses)

- Want to make images
- Pinhole camera models the geometry of perspective projection
- Lenses make it work in practice
- Models for lenses
  - –Thin lens, spherical surfaces, first order optics
  - –Thick lens, higher-order optics, vignetting.

Perspective projection

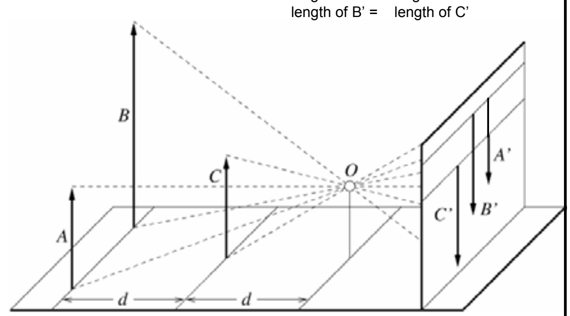
- Abstract camera model - box with a small hole in it
- In an ideal pinhole camera everything is in focus



Forsyth&Ponce

Distant objects are smaller

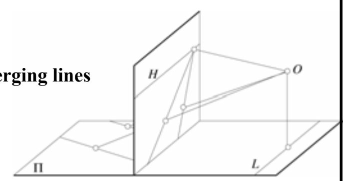
length of B = 2*length of C
length of B' = length of C'



Forsyth&Ponce

Effect of projection

- **Points go to points**
- **Lines go to lines**
- **Planes go to a half plane**
- **Parallel lines go to converging lines**
- **Polygons go to polygons**



- **Degenerate cases:**

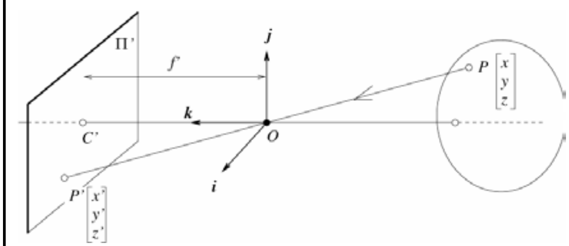
  Line through the pinhole go to points

  Planes through the pinhole go to a line

  Parallels parallel to the image plane stay parallel

  Planes parallel to the image plane goes to full planes

The equation of projection



The equation of projection

- **Cartesian coordinates:**

  We have, by similar triangles, that

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z}, -f)$$

Ignore the third coordinate, and get

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogenous coordinates

- **Add an extra coordinate and use an equivalence relation**
- **for 2D**

  equivalence relation *k*(x,y,z)* is the same as *(x,y,z)*
- **for 3D**

  equivalence relation *k*(x,y,z,t)* is the same as *(x,y,z,t)*
- **Basic notion**

  Possible to represent points "at infinity"
  - Where parallel lines intersect
  - Where parallel planes intersect

  Possible to write the action of a perspective camera as a matrix

The camera matrix

- **Turn previous expression into HC's**

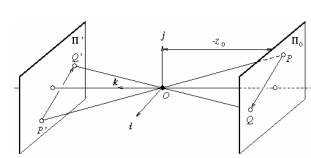
  HC's for 3D point are *(x,y,z,t)*

  HC's for point in image are *(u,v,w)*

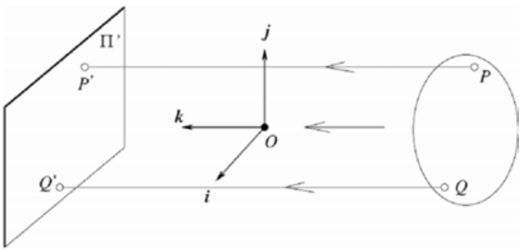
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

Weak perspective

- **Issue**
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong



Orthographic projection



Camera parameters

- **Issue**

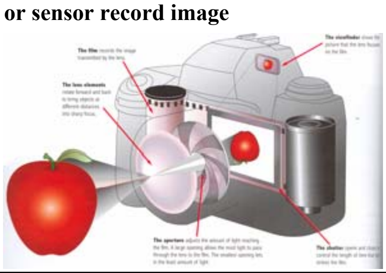
  camera may not be at the origin, looking down the z-axis
  - extrinsic parameters

  one unit in camera coordinates may not be the same as one unit in world coordinates
  - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

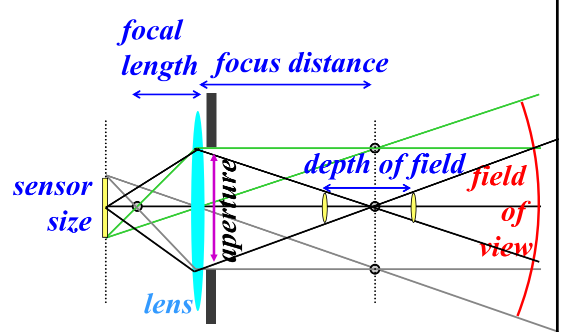
$$\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}\text{Transformation}\\ \text{representing}\\ \text{intrinsic parameters}\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix}\begin{pmatrix}\text{Transformation}\\ \text{representing}\\ \text{extrinsic parameters}\end{pmatrix}\begin{pmatrix}x\\y\\z\\t\end{pmatrix}$$

Camera Overview

- **Lens and viewpoint determine perspective**
- **Aperture and shutter speed determine exposure**
- **Aperture and other effects determine depth of field**
- **Film or sensor record image**



Quantities



- **Focal length (in mm)**

  Determines the field of view.
  wide angle (<30mm) to telephoto (>100mm)
- **Focusing distance**

  Which distance in the scene is sharp
- **Depth of field**

  Given tolerance, zone around the focus distance that is sharp
- **Aperture (in f number)**

  Ratio of used diameter and focal lens.
  Number under the divider ➔ small number = large aperture
  (e.g. f/2.8 is a large aperture, f/16 is a small aperture)
- **Shutter speed (in fraction of a second)**

  Reciprocity relates shutter speed and aperture
- **Sensitivity (in ISO)**

  Linear effect on exposure
  100 ISO is for bright scenes, ISO 1600 is for dark scenes

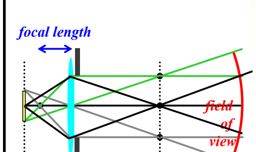
Focal length

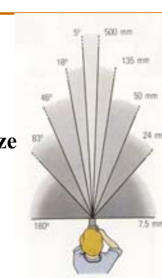
<30mm: wide angle

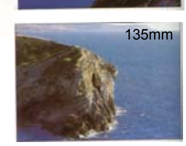
50mm: standard

>100mm telephoto

Affected by sensor size (crop factor)

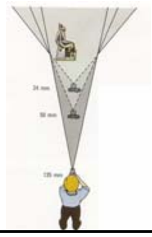






Perspective vs. viewpc

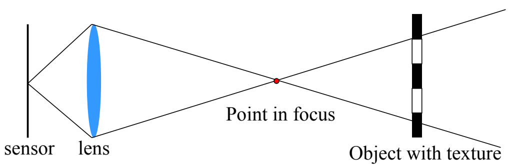
- **Telephoto makes it easier to select background (a small change in viewpoint is a big change in background.**



Depth of field

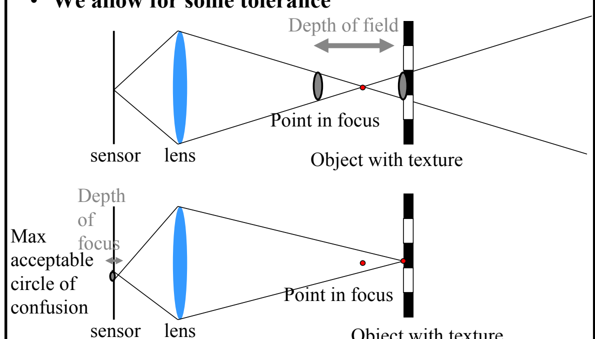
- **The bigger the aperture (small f number), the shallower the DoF**
  - Just think Gaussian blur: bigger kernel ➔ more blurry
  - This is the advantage of lenses with large maximal aperture: they can blur the background more
- **The closer the focus, the smaller the DoF**
- **Focal length has a more complex effect on DoF**
  - Distant background more blurry with telephoto
  - Near the focus plane, depth of field only depends on image size
- **Hyperfocal distance:**
  - Closest focusing distance for which the depth of field includes infinity
  - The largest depth of field one can achieve.
  - Depends on aperture.

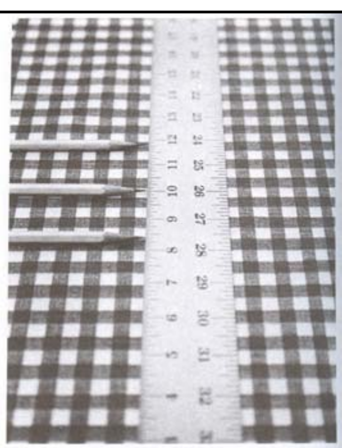
Depth of field



Depth of field

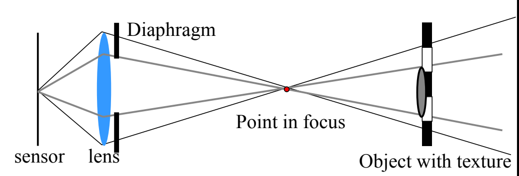
- **We allow for some tolerance**





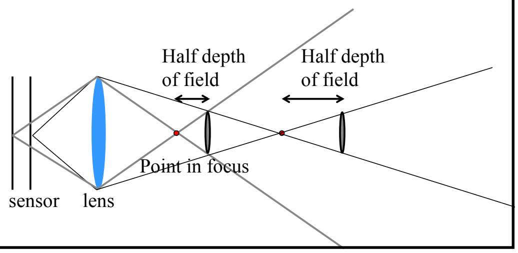
Depth of field

- **What happens when we close the aperture by two stop?**
  - Aperture diameter is divided by two
  - Depth of field is doubled



Depth of field & focusing distance

- **What happens when we divide focusing distance by two?**
  Similar triangles => divided by two as well



Exposure

- **Aperture (f number)**
  Expressed as ratio between focal length and aperture diameter:
  diameter = f / <f number>
  f/2.0, f/2.8, f/4.0, f/5.6, f/8.0, f/11, f/16 (factor of sqrt (2))
  Small f number means large aperture
  Main effect: depth of field
  A good standard lens has max aperture f/1.8.
  A cheap zoom has max aperture f/3.5
- **Shutter speed**
  In fraction of a second
  1/30, 1/60, 1/125, 1/250, 1/500 (factor of 2)
  Main effect: motion blur
  A human can usually hand-hold up to 1/f seconds, where f is focal length
- **Sensitivity**
  Gain applied to sensor
  In ISO, bigger number, more sensitive (100, 200, 400, 800, 1600)
  Main effect: sensor noise

Reciprocity between these three numbers:
for a given exposure, one has two degrees of freedom.

Recap

- **Pinhole is the simplest model of image formation**
- **Lenses gather more light**
  But get only one plane focused
  Focus by moving sensor/film
  Cannot focus infinitely close
- **Focal length determines field of view**
  From wide angle to telephoto
  Depends on sensor size

Next Time

Camera Calibration and Radiometry